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EUROPEAN PUT OPTION PRICING MODEL WITH GRAM-CHARLIER EXPANSION IN THIRD MOMENTS

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ABSTRACT

The Black-Scholes model is one of the most popular and widely applied option pricing models in both academic and practical contexts developed by Black and Scholes (1973). The practical assumption in the Black-Scholes model is stock return following the normal distribution with constant volatility. However, many stock returns are not normally distributed, so should consider the skewness and kurtosis of the stock return. This developmental model adapts the Gram-Charlier expansion to adapt skewness and kurtosis to the Black-Scholes formula. Approximation method used is an alternative approach with Hermite polynomial. The observed stocks are SPG, C, and AXP by taking stock price data from November 11, 2016 to November 11, 2017 with maturity date at January 18, 2019 and interest rate (r) of 1,25%. After comparing the average MSE of both models, found that the third moment Gram-Charlier expansion is better than the Black-Scholes model in modeling SPG, C, and TSLA stock prices.

Keywords: *Stock Option, Black-scholes, Gram-Charlier Expansion, Hermite Polynomial*

INTRODUCTION

Option is a contract or agreement between two parties, where the first party as a buyer has the right to buy or sell from a second party that is the seller of a particular asset at a specified price and time. The Black-Scholes model is one of the popular option pricing models developed by Black and Scholes (1973). The Black-Scholes model assumes that stock return follows the normal distribution (skewness 0 and kurtosis 3) with constant volatility. Hull (1993) and Nattenburg (1994) point out that stock returns exhibit nonnormal skewness and kurtosis and that volatility skews are a consequence of empirical violations of the normality assumption.

Corrado and Su (1996) develop a method to incorporate option price adjustments for nonnormal skewness and kurtosis in an expanded Black–Scholes option pricing formula. Their method adapts a Gram–Charlier series expansion of the standard normal density function to yield an option price formula that is the sum of a Black–Scholes option price plus adjustment terms for nonnormal skewness and kurtosis.

In this paper, it will be explained that the pricing of options with returns not normally distributed can be found by considering the skewness (μ_3) of the return. This model adapts the Gram-Charlier expansion to provide skewness and kurtosis adjustments to the Black-Scholes formula. Approximation method used is an alternative approach with Hermite polynomial.

Black-Scholes Model

The formula of the price of put option Black-Scholes model as present value of expected benefit of European put option.

$$\begin{aligned} P_{BS} &= e^{-rT}E[\max(K - S_T, 0)] \\ &= Ke^{-rT}N(-d_2) - S_0N(-d_1). \end{aligned}$$

Hermite Polynomials

Given density function of standard normal distribution $n(z)$ $D = \frac{d}{dz}$ as differentiation operator. Hermite Polynomials are a classical orthogonal polynomial sequence at interval (-∞,∞), can be defined from Rodrigues Formula as follows:

$$H_n(z)n(z) = (-D)^n n(z)$$
$$= (-1)^n \frac{d^n}{dz^n} n(z).$$

(2)

Then we get,

$$H_n(z) = (-1)^n e^{\frac{z^2}{2}} \frac{d^n}{dz^n}\left(e^{-\frac{z^2}{2}}\right).$$

For $n = 3$, obtained :

$H_3(z)n(z) = (-D)^3 n(z) = (z^3 - 3z)n(z) \quad \rightarrow \; H_3(z) = (z^3 - 3z)$

The properties of orthogonal polynomial Hermite i.e:

$$\int_{-\infty}^{\infty} H_m(z) H_n(z) n(z)\, dz = \begin{cases} 0 & \text{,jika m} \neq \text{n} \\ \text{m!} & \text{,jika m = n} \end{cases}$$

(3)

***Gram-Charlier* Expansion**

Density function of Gram-Charlier expansion as follows:

$$g(z) = \sum_{n=0}^{n/2} c_n H_n(z)n(z) \qquad (4\ )$$

Where $n(z)$ is density function of standard distribution normal, $H_n(z)$ nth order of polynomial Hermite. Coefficient c_n on equation (4) come from Polynomial Hermite. If equation (4) to side multiplied by $H_m(z)$ then, integrated from the boundary -∞ until ∞ then obtained:

$$\int_{-\infty}^{\infty} g(z)H_m(z)dz = c_0 \int_{-\infty}^{\infty} H_0(z)H_m(z)n(z)dz + c_1 \int_{-\infty}^{\infty} H_1(z)H_m(z)n(z)dz + \cdots +$$
$$c_m \int_{-\infty}^{\infty} H_m(z)H_m(z)n(z)dz + c_{m+1} \int_{-\infty}^{\infty} H_{m+1}(z)H_m(z)n(z)dz + \cdots$$

Using the properties of orthogonal polynomial Hermite in equation (3) obtained:

$$\int_{-\infty}^{\infty} g(z)H_m(z)dz = 0 + 0 + \cdots + c_m \int_{-\infty}^{\infty} H_m(z)H_m(z)n(z)dz + ...$$
$$= c_m \int_{-\infty}^{\infty} H_m(z)H_m(z)n(z)dz$$
$$= c_m m!$$

We get: $c_m = \frac{1}{m!} \int_{-\infty}^{\infty} g(z) H_m(z)\, dz$

From the above equation obtained the value of c_m as follows:

$c_0 = \frac{1}{0!} \int_{-\infty}^{\infty} g(z)H_0(z)dz = \int_{-\infty}^{\infty} g(z)\, dz = 1$

$c_1 = \frac{1}{1!} \int_{-\infty}^{\infty} g(z)H_1(z)dz = \int_{-\infty}^{\infty} g(z)\, zdz = E(z) = \mu_1$

$c_2 = \frac{1}{2!} \int_{-\infty}^{\infty} g(z)H_2(z)dz = \frac{1}{2!} \int_{-\infty}^{\infty} g(z)(z^2 - 1)dz = \frac{1}{2!}[\mu_2 - 1]$

$c_3 = \frac{1}{3!} \int_{-\infty}^{\infty} g(z)H_3(z)dz = \frac{1}{3!} \int_{-\infty}^{\infty} g(z)(z^3 - 3z)dz = \frac{1}{3!}[\mu_3 - 3\mu_1]$

...

Gram-Charlier expansion is obtained as follows:

$$g(z) = \sum_{n=0}^{\infty} c_n H_n(z)n(z) = n(z) \sum_{n=0}^{\infty} c_n H_n(z)$$
$$= n(z)[c_0 H_0(z) + c_1 H_1(z) + c_2 H_2(z) + \cdots]$$

$$= n(z)[H_0(z) + \mu_1 H_1(z) + \frac{1}{2!}[\mu_2 - 1]H_2(z) + \frac{1}{3!}[\mu_3 - 3\mu_1]H_3(z) + \cdots$$

Since the moment used only until the third moment then

$$g(z) = n(z)[H_0(z) + \mu_1 H_1(z) + \frac{1}{2!}[\mu_2 - 1]H_2(z) + \frac{1}{3!}[\mu_3 - 3\mu_1]H_3(z)$$

In addition, given that z is standard normal distribution, then $E(z) = \mu_1 = 0$ and $E(z^2) = \mu_2 = 1$ so that *Gram-Charlier* expansion above become:

$$g(z) = n(z)[1 + \frac{\mu_3}{3!} H_3(z)]$$

European Put Option Price by Gram-Charlier Expansion

With substituting $S_T = e^{z\sigma\sqrt{T}+m}$ and $-d_2 = \dfrac{\ln K - m}{\sigma\sqrt{T}}$ then

$$P_3 = e^{-rT} \int_0^K (K - S_T) g(S_T) d(S_T)$$

$$= e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(K - e^{z\sigma\sqrt{T}+m}\right) n(z) \left(1 + \frac{\mu_3}{3!} H_3(z)\right) dz$$

$$= e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(K - e^{z\sigma\sqrt{T}+m}\right) n(z) dz + e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \Big(K -$$

$$e^{z\sigma\sqrt{T}+m}\Big) n(z) \left(\frac{\mu_3}{3!} H_3(z)\right) dz$$

$$= P_{BS} + \frac{\mu_3}{3!} e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(K - e^{z\sigma\sqrt{T}+m}\right) n(z) H_3(z) dz$$

$$let\ I_3 = e^{-rT} \int_0^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(K - e^{z\sigma\sqrt{T}+m}\right) n(z) H_3(z) dz$$

$$I_3 = e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(K - e^{z\sigma\sqrt{T}+m}\right) n(z) H_3(z) dz$$

$$= -e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(K - e^{z\sigma\sqrt{T}+m}\right) \frac{d}{dz}\left(\frac{d^2 n(z)}{dz^2}\right) dz$$

$$let\!: u = K - e^{z\sigma\sqrt{T}+m}, du = -\sigma\sqrt{T} e^{z\sigma\sqrt{T}+m} dz, dv = \frac{d}{dz}\left(\frac{d^2 n(z)}{dz^2}\right) dz, v = \left(\frac{d^2 n(z)}{dz^2}\right)$$

$$I_3 = -e^{-rT} \left(\left(K - e^{z\sigma\sqrt{T}+m}\right) \left(\frac{d^2 n(z)}{dz^2}\right) \Big]_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} - \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(\frac{d^2 n(z)}{dz^2}\right) \left(-\sigma\sqrt{T} e^{z\sigma\sqrt{T}+m}\right) dz \right)$$

$$= -e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(\sigma\sqrt{T} e^{z\sigma\sqrt{T}+m}\right) \left(\frac{d^2 n(z)}{dz^2}\right) dz$$

$$let\!: u = \sigma\sqrt{T} e^{z\sigma\sqrt{T}+m}, du = \sigma^2 T e^{z\sigma\sqrt{T}+m}, dv = \frac{d^2 n(z)}{dz^2} dz, v = \frac{dn(z)}{dz}$$

$$I_3 = -e^{-rT} \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(\sigma\sqrt{T} e^{z\sigma\sqrt{T}+m}\right) \left(\frac{d^2 n(z)}{dz^2}\right) dz$$

$$= -e^{-rT} \sigma\sqrt{T} K \left(-\frac{\ln K - m}{\sigma\sqrt{T}}\right) n\left(\frac{\ln K - m}{\sigma\sqrt{T}}\right) + e^{-rT} \sigma^2 T \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(e^{z\sigma\sqrt{T}+m}\right) \left(\frac{dn(z)}{dz}\right) dz$$

$$let\!: u = e^{z\sigma\sqrt{T}+m}, du = \sigma\sqrt{T} e^{z\sigma\sqrt{T}+m} dz, dv = \left(\frac{dn(z)}{dz}\right) dz, v = n(z)$$

$$\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} \left(e^{z\sigma\sqrt{T}+m}\right)\left(\frac{dn(z)}{dz}\right)dz = e^{z\sigma\sqrt{T}+m}n(z)\Big]_0^{\frac{lnK-m}{\sigma\sqrt{T}}} - \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} n(z)\sigma\sqrt{T}e^{z\sigma\sqrt{T}+m}dz$$

$$= K\,n\left(\frac{ln\,K-m}{\sigma\sqrt{T}}\right) - \sigma\sqrt{T}\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} e^{z\sigma\sqrt{T}+m}n(z)dz$$

$I_3 =$

$$-e^{-rT}\sigma\sqrt{T}K\left(-\frac{\ln K-m}{\sigma\sqrt{T}}\right)n\left(\frac{\ln K-m}{\sigma\sqrt{T}}\right)e^{-rT}\sigma^2 T\left(K\,n\left(\frac{lnK-m}{\sigma\sqrt{T}}\right)\sigma\sqrt{T}\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} e^{z\sigma\sqrt{T}+m}n(z)dz\right)$$

$=$

$$-e^{-rT}\sigma\sqrt{T}K\left(-\frac{\ln K-m}{\sigma\sqrt{T}}\right)n\left(\frac{\ln K-m}{\sigma\sqrt{T}}\right)e^{-rT}\sigma^2 TK\,n\left(\frac{lnK-m}{\sigma\sqrt{T}}\right)e^{m-rT}\left(\sigma\sqrt{T}\right)^3\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} e^{z\sigma\sqrt{T}}n(z)dz.$$

$$\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} e^{z\sigma\sqrt{T}}n(z)dz = \int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} e^{z\sigma\sqrt{T}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz$$

$$= \frac{1}{\sqrt{2\pi}}e^{\frac{1}{2}\sigma^2 T}\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} e^{-\frac{1}{2}\left(z-\sigma\sqrt{T}\right)^2}dz$$

We assumed that $y = z - \sigma\sqrt{T}, dy = dz$.

$$\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}} e^{z\sigma\sqrt{T}}n(z)dz = e^{\frac{1}{2}\sigma^2 T}\int_{-\infty}^{\frac{lnK-m}{\sigma\sqrt{T}}-\sigma\sqrt{T}} \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dz$$

$$= e^{\frac{1}{2}\sigma^2 T}N\left(\frac{lnK-m}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right)$$

$$I_3 = -e^{-rT}\sigma\sqrt{T}K\left(-\frac{\ln K-m}{\sigma\sqrt{T}}\right)n\left(\frac{\ln K-m}{\sigma\sqrt{T}}\right) + e^{-rT}\sigma^2 TK\,n\left(\frac{lnK-m}{\sigma\sqrt{T}}\right) - e^{m-rT+\frac{1}{2}\sigma^2 T}\left(\sigma\sqrt{T}\right)^3 N\left(\frac{lnK-m}{\sigma\sqrt{T}} - \sigma\sqrt{T}\right)$$

$for\ m = \ln S_0 + \left(r - \frac{1}{2}\sigma^2\right)T,$

$d_2 = \frac{\ln K-m}{\sigma\sqrt{T}},$

$d_2 = d_1 - \sigma\sqrt{T}, n(-d_2) = n(d_2), and\ e^{-rT}K\,n(d_2) = S_0 n(d_1).$

We have

$I_3 = -\sigma\sqrt{T}S_0\,n(d_1)\left(d_1 - \sigma\sqrt{T}\right) + \sigma^2 TS_0\,n(d_1) - e^{\ln S_0+\left(r-\frac{1}{2}\sigma^2\right)T-rT+\frac{1}{2}\sigma^2 T}\left(\sigma\sqrt{T}\right)^3 N(-d_1)$

$I_3 = \sigma\sqrt{T}S_0\ \left(n(d_1)\left(2\sigma\sqrt{T} - d_1\right) - \sigma^2 T\,N(-d_1)\right)$

Thus,

$$P_{GC-3} = P_{BS} + \mu_3 Q_3$$

With,

$\mu_3 = Skewness$

$$Q_3 = \frac{S_0\, \sigma\sqrt{T}\left(n(d_1)\left(2\sigma\sqrt{T} - d_1\right) - \sigma^2 T\, N(-d_1)\right)}{3!}$$

$$d_1 = \frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Case Studies

In this case study we will calculate the option price of European type of expansion model of Gram-Charlier and Black-Scholes model. After that, the calculations from both models will be compared with the option price in the market. Interest rate risk (r) = 1.25% (Data from the Federal Fund Rate). The stock data observed were Simon Property Group, Inc. (SPG), Citigroup Inc. (C), and American Express Company (AXP).The data taken is daily stock price data for 1 year (Nov 11, 2016 - Nov 11, 2017). Then calculate the Log-return value with the formula:

$$R_T = \ln \frac{S_T}{S_{T-1}}$$

From Log-return data of the three stocks above, the following data are obtained:

	STOCK		
	SPG	**C**	**AXP**
Stock Price (S_0)	$163.75	$72.25	$93.52
Average of Return	-0.000425627	0.001184114	0.001799112
Volatility (σ)	20.65%	18.26%	21.75%
Risk free rate (r)	1.25%	1.25%	1.25%
Time (T)	0.277777778	0.277777778	0.277777778
Skewness (μ_3)	-0.236470618	0.01097949	7.791851308

After calculation using Microsoft Excel software, the price of put option is as follows:

<u>Simon Property Group, Inc. Put Price</u>

Below is the option pricing details for Simon Property Group, Inc.:

K	Q3	P-Market	P-BS	P-GC-3	MSE P-BS	MSE P-GC
130.00	-0.20684	7.80	0.088435	0.137347	59.468232	58.716256
135.00	-0.34603	9.70	0.226925	0.308752	89.739145	88.195534
140.00	-0.48606	10.01	0.513443	0.628382	90.184600	88.014758
145.00	-0.57021	13.55	1.040078	1.174915	156.498156	153.142723
155.00	-0.38118	18.00	3.230104	3.320243	218.149830	215.495275
160.00	-0.10583	20.70	5.06915	5.094177	244.323461	243.541716
165.00	0.221035	20.60	7.464904	7.412636	172.530745	173.906573
180.00	0.845123	28.88	17.72195	17.5221	124.502077	129.001816
185.00	0.838926	37.20	21.93284	21.73446	233.086142	239.182937
190.00	0.750631	36.93	26.40133	26.22383	110.852805	114.622032
195.00	0.616546	51.85	31.05433	30.90854	432.459758	438.544823

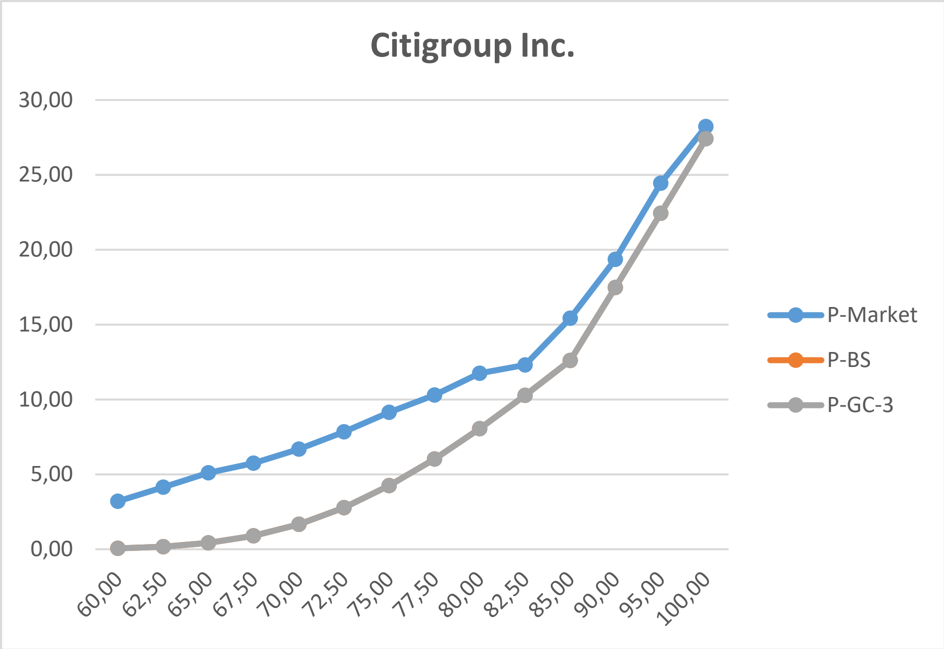
210.00	0.220275	44.50	45.60635	45.55426	1.224006	1.111463
220.00	0.071928	52.43	55.51064	55.49363	9.490319	9.385812
280.00	-0.03513	103.60	115.2795	115.2878	136.409923	136.604040
				Mean	148.494229	149.247554

Based on the Mean Square Error (MSE) value above, the Gram-Charlier model option price is better than the Black-Scholes model because it has a smaller MSE. In other words, the Gram-Charlier model is closer to the price of the market option than the Black-Scholes model option price.

Citigroup Inc. Put Price

Below is the option pricing details for Citigroup Inc.:

K	Q3	P-Market	P-BS	P-GC-3	MSE P-BS	MSE P-GC
60.00	-0.11092	3.20	0.058643	0.057425	9.868125	9.875778
62.50	-0.18303	4.15	0.171469	0.16946	15.828706	15.844700
65.00	-0.22873	5.10	0.421108	0.418597	21.892030	21.915537
67.50	-0.20465	5.75	0.890471	0.888224	23.615022	23.636865
70.00	-0.09725	6.68	1.657794	1.656726	25.222558	25.233283
72.50	0.06141	7.83	2.772066	2.77274	25.582698	25.575878
75.00	0.212498	9.15	4.238431	4.240764	24.123507	24.100594
77.50	0.306456	10.30	6.020333	6.023698	18.315551	18.286762
80.00	0.326593	11.75	8.054886	8.058471	13.653871	13.627383
82.50	0.287744	12.30	10.27208	10.27524	4.112454	4.099650
85.00	0.219147	15.42	12.6096	12.612	7.898357	7.884839
90.00	0.088025	19.35	17.47113	17.4721	3.530143	3.526512
95.00	0.019539	24.45	22.42659	22.42681	4.094179	4.093311
100.00	-0.00361	28.22	27.40427	27.40423	0.665422	0.665487
				Mean	14.171616	14.169041



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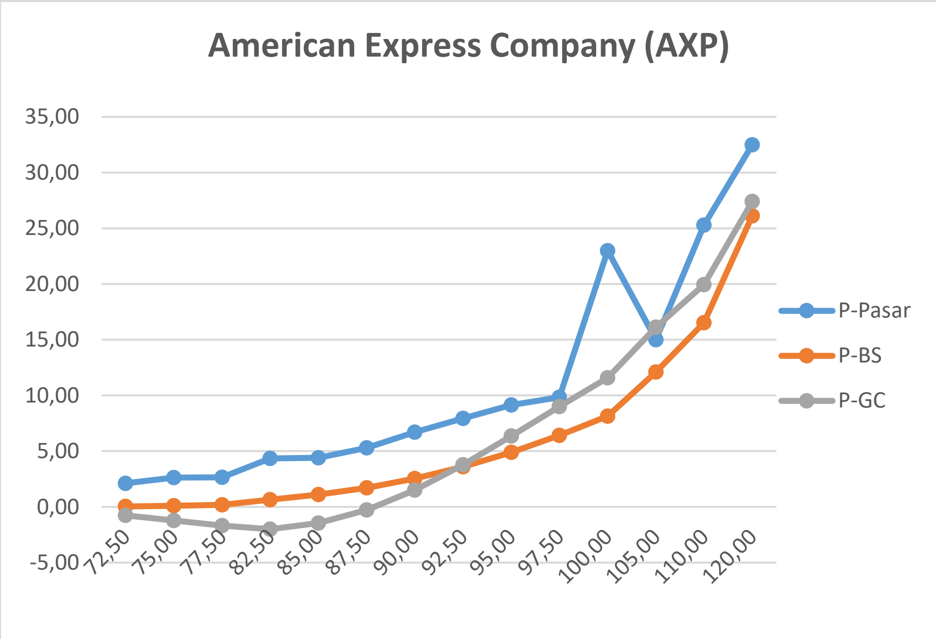
American Express Company Put Price

Below is the option pricing details for American Express Company:

K	Q3	P-Pasar	P-BS	P-GC	MSE P-BS	MSE P-GC
72.50	-0.103327267	2.12	0.039645151	-0.765465546	4.327876	8.325911
75.00	-0.168081415	2.64	0.091372201	-1.21829319	6.495504	14.886426
77.50	-0.241223451	2.65	0.1914317	-1.688145561	6.044558	18.819507
82.50	-0.340471886	4.35	0.656427768	-1.996478541	13.642476	40.277790
85.00	-0.326795837	4.40	1.093153054	-1.453191517	10.935237	34.259851
87.50	-0.256145674	5.30	1.713527396	-0.282321608	12.862786	31.162315
90.00	-0.13379017	6.70	2.545543529	1.503070421	17.259509	27.008077
92.50	0.022158365	7.93	3.606211296	3.778865984	18.695149	17.231914
95.00	0.186120964	9.14	4.899640372	6.349867245	17.980650	7.784841
97.50	0.332332548	9.85	6.417358504	9.006844301	11.783028	0.710912
100.00	0.441193101	23.00	8.140514422	11.57822547	220.804312	130.456934
105.00	0.517034678	15.00	12.09616859	16.12482592	8.432237	1.265233
110.00	0.437992723	25.30	16.53711111	19.94988529	76.788222	28.623727
120.00	0.163728965	32.50	26.13312485	27.40887659	40.537099	25.919538
				Mean	33.327760	27.623784



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Conclution

Based on the results of the discussion and case studies, the following conclusions can be obtained:

1. The pricing of European type of put options with Gram-Charlier expansion is obtained by a hermite polynomial approach. Put option price obtained is the Black-Scholes put option price plus the equation relating to abnormal skewness and kurtosis.
2. The option price with Gram-Charlier expansion is influenced by several factors: initial stock price (S_0), strike price (K), interest rate risk (r), maturity (T), volatility (σ), skewness and kurtosis.
3. Return of stocks with skewness and kurtosis values near 0 and 3, will result in the price of the Gram-Charlier Expansion option approaching the Black-Scholes model option price.
4. In SPG and C put price the Gram-Charlier model option price is much better than the option price with the Black-Scholes model because it has a smaller MSE. In other words, the Gram-Charlier model for this stock is closer to the price of the market option compared to the Black-Scholes model option price, but not for AXP stock.

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